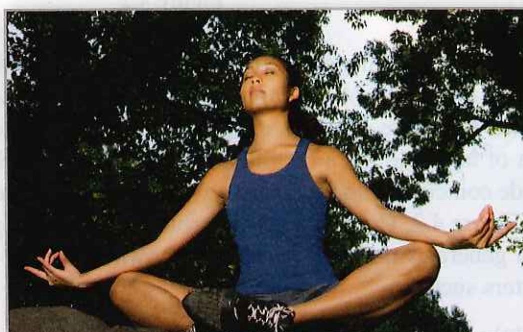
4 Trigonometry

Television Coverage *(Exercise 84, page 319)*

Waterslide Design
(Exercise 32, page 337)



Respiratory Cycle *(Exercise 88, page 308)*

Meteorology
(Exercise 99, page 298)

Skateboarding *(Example 10, page 285)*

Clockwise from top left, ariadna de raadt/Shutterstock.com; Alexey Bykov/Shutterstock.com; tusharkoley/Shutterstock.com; Vladimir Ivanovich Danilov/Shutterstock; AISPIX by Image Source/Shutterstock.com

4.1 Radian and Degree Measure

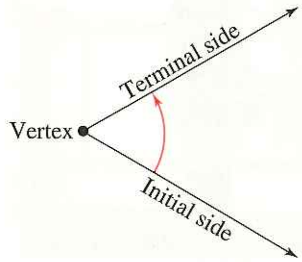
Angles can help you model and solve real-life problems. For instance, in Exercise 68 on page 271, you will use angles to find the speed of a bicycle.

- **Describe angles.**
- **Use radian measure.**
- **Use degree measure.**
- **Use angles to model and solve real-life problems.**

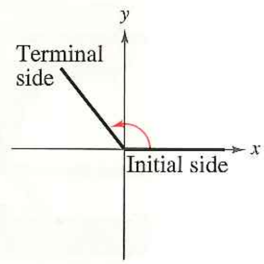
Angles

As derived from the Greek language, the word **trigonometry** means "measurement of triangles." Initially, trigonometry dealt with relationships among the sides and angles of triangles and was used in the development of astronomy, navigation, and surveying. With the development of calculus and the physical sciences in the 17th century, a different perspective arose—one that viewed the classic trigonometric relationships as *functions* with the set of real numbers as their domains. Consequently, the applications of trigonometry expanded to include a vast number of physical phenomena, such as sound waves, planetary orbits, vibrating strings, pendulums, and orbits of atomic particles.

This text incorporates *both* perspectives, starting with angles and their measures.



Angle
Figure 4.1



Angle in standard position
Figure 4.2

An **angle** is determined by rotating a ray (half-line) about its endpoint. The starting position of the ray is the **initial side** of the angle, and the position after rotation is the **terminal side,** as shown in Figure 4.1. The endpoint of the ray is the **vertex** of the angle. This perception of an angle fits a coordinate system in which the origin is the vertex and the initial side coincides with the positive x-axis. Such an angle is in **standard position,** as shown in Figure 4.2. Counterclockwise rotation generates **positive angles** and clockwise rotation generates **negative angles,** as shown in Figure 4.3. Angles are labeled with Greek letters such as

α (alpha), β (beta), and θ (theta)

as well as uppercase letters such as

A, B, and C.

In Figure 4.4, note that angles α and β have the same initial and terminal sides. Such angles are **coterminal.**

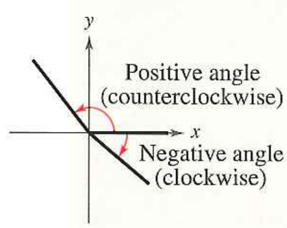
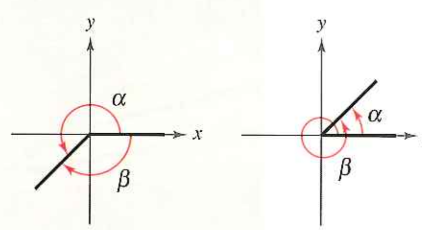
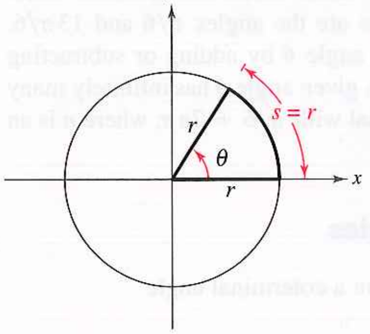


Figure 4.3



Coterminal angles
Figure 4.4



Arc length = radius when $\theta = 1$ radian.
Figure 4.5

Radian Measure

You determine the **measure of an angle** by the amount of rotation from the initial side to the terminal side. One way to measure angles is in *radians*. This type of measure is especially useful in calculus. To define a radian, you can use a **central angle** of a circle, one whose vertex is the center of the circle, as shown in Figure 4.5.

Definition of Radian

One **radian** is the measure of a central angle θ that intercepts an arc s equal in length to the radius r of the circle. See Figure 4.5. Algebraically, this means that

$$\theta = \frac{s}{r}$$

where θ is measured in radians. (Note that $\theta = 1$ when $s = r$.)

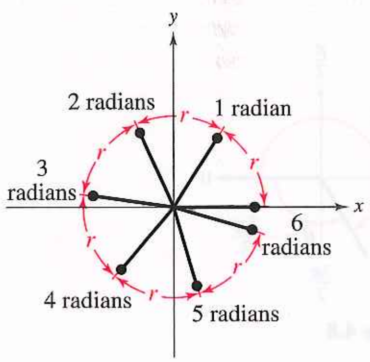


Figure 4.6

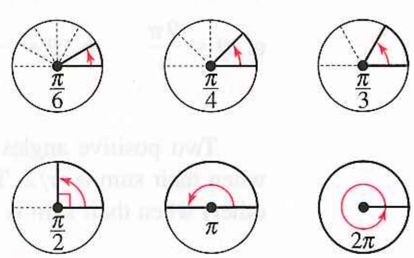
Because the circumference of a circle is $2\pi r$ units, it follows that a central angle of one full revolution (counterclockwise) corresponds to an arc length of $s = 2\pi r$. Moreover, because $2\pi \approx 6.28$, there are just over six radius lengths in a full circle, as shown in Figure 4.6. Because the units of measure for s and r are the same, the ratio s/r has no units—it is a real number.

Because the measure of an angle of one full revolution is $s/r = 2\pi r/r = 2\pi$ radians, you can obtain the following.

$\frac{1}{2}$ revolution $= \frac{2\pi}{2} = \pi$ radians $\qquad$ $\frac{1}{4}$ revolution $= \frac{2\pi}{4} = \frac{\pi}{2}$ radians

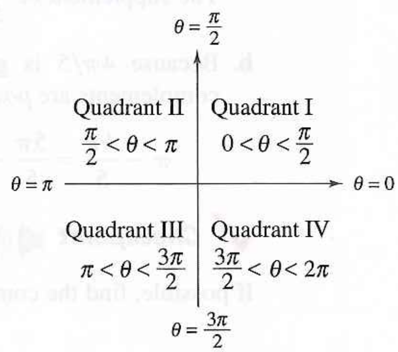
$\frac{1}{6}$ revolution $= \frac{2\pi}{6} = \frac{\pi}{3}$ radians

These and other common angles are shown below.



REMARK The phrase "the terminal side of θ lies in a quadrant" is often abbreviated by the phrase "θ lies in a quadrant." The terminal sides of the "quadrant angles" 0, $\pi/2$, π, and $3\pi/2$ do not lie within quadrants.

Recall that the four quadrants in a coordinate system are numbered I, II, III, and IV. The figure below shows which angles between 0 and 2π lie in each of the four quadrants. Note that angles between 0 and $\pi/2$ are **acute** angles and angles between $\pi/2$ and π are **obtuse** angles.



Two angles are coterminal when they have the same initial and terminal sides. For instance, the angles 0 and 2π are coterminal, as are the angles $\pi/6$ and $13\pi/6$. You can find an angle that is coterminal to a given angle θ by adding or subtracting 2π (one revolution), as demonstrated in Example 1. A given angle θ has infinitely many coterminal angles. For instance, $\theta = \pi/6$ is coterminal with $\pi/6 + 2n\pi$, where n is an integer.

ALGEBRA HELP You can review operations involving fractions in Appendix A.1.

EXAMPLE 1 Finding Coterminal Angles

a. For the positive angle $13\pi/6$, subtract 2π to obtain a coterminal angle

$$\frac{13\pi}{6} - 2\pi = \frac{\pi}{6}.$$ See Figure 4.7.

b. For the negative angle $-2\pi/3$, add 2π to obtain a coterminal angle

$$-\frac{2\pi}{3} + 2\pi = \frac{4\pi}{3}.$$ See Figure 4.8.

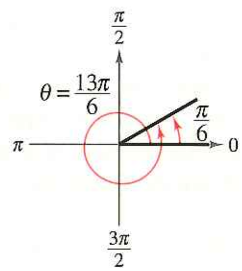


Figure 4.7

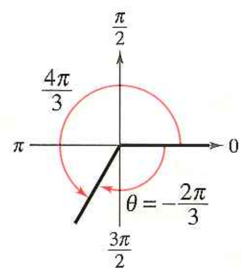
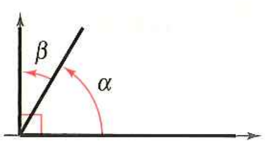


Figure 4.8

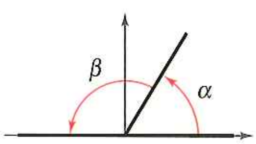
✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com.*

Determine two coterminal angles (one positive and one negative) for each angle.

a. $\theta = \dfrac{9\pi}{4}$ **b.** $\theta = -\dfrac{\pi}{3}$



Complementary angles



Supplementary angles
Figure 4.9

Two positive angles α and β are **complementary** (complements of each other) when their sum is $\pi/2$. Two positive angles are **supplementary** (supplements of each other) when their sum is π. See Figure 4.9.

EXAMPLE 2 Complementary and Supplementary Angles

a. The complement of $\dfrac{2\pi}{5}$ is $\dfrac{\pi}{2} - \dfrac{2\pi}{5} = \dfrac{5\pi}{10} - \dfrac{4\pi}{10} = \dfrac{\pi}{10}$.

The supplement of $\dfrac{2\pi}{5}$ is $\pi - \dfrac{2\pi}{5} = \dfrac{5\pi}{5} - \dfrac{2\pi}{5} = \dfrac{3\pi}{5}$.

b. Because $4\pi/5$ is greater than $\pi/2$, it has no complement. (Remember that complements are *positive* angles.) The supplement of $4\pi/5$ is

$$\pi - \frac{4\pi}{5} = \frac{5\pi}{5} - \frac{4\pi}{5} = \frac{\pi}{5}.$$

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com.*

If possible, find the complement and the supplement of (a) $\pi/6$ and (b) $5\pi/6$.

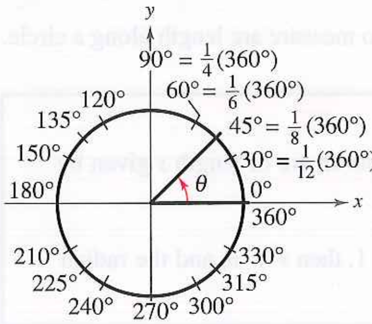


Figure 4.10

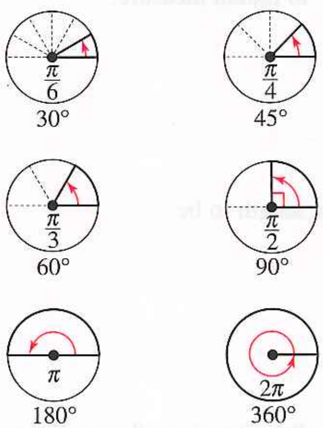


Figure 4.11

TECHNOLOGY

With calculators, it is convenient to use *decimal* degrees to denote fractional parts of degrees. Historically, however, fractional parts of degrees were expressed in *minutes* and *seconds*, using the prime (′) and double prime (″) notations, respectively. That is,

$$1' = \text{one minute} = \tfrac{1}{60}(1°)$$
$$1'' = \text{one second} = \tfrac{1}{3600}(1°).$$

Consequently, an angle of 64 degrees, 32 minutes, and 47 seconds is represented by $\theta = 64° \, 32' \, 47''$. Many calculators have special keys for converting an angle in degrees, minutes, and seconds (D° M′ S″) to decimal degree form, and vice versa.

Degree Measure

A second way to measure angles is in **degrees,** denoted by the symbol °. A measure of one degree (1°) is equivalent to a rotation of $\frac{1}{360}$ of a complete revolution about the vertex. To measure angles, it is convenient to mark degrees on the circumference of a circle, as shown in Figure 4.10. So, a full revolution (counterclockwise) corresponds to 360°, a half revolution to 180°, a quarter revolution to 90°, and so on.

Because 2π radians corresponds to one complete revolution, degrees and radians are related by the equations

$$360° = 2\pi \text{ rad} \quad \text{and} \quad 180° = \pi \text{ rad}.$$

From the latter equation, you obtain

$$1° = \frac{\pi}{180} \text{ rad} \quad \text{and} \quad 1 \text{ rad} = \left(\frac{180}{\pi}\right)^{\circ}$$

which lead to the following conversion rules.

Conversions Between Degrees and Radians

1. To convert degrees to radians, multiply degrees by $\dfrac{\pi \text{ rad}}{180°}$.

2. To convert radians to degrees, multiply radians by $\dfrac{180°}{\pi \text{ rad}}$.

To apply these two conversion rules, use the basic relationship $\pi \text{ rad} = 180°$. (See Figure 4.11.)

When no units of angle measure are specified, *radian measure is implied.* For instance, $\theta = 2$ implies that $\theta = 2$ radians.

EXAMPLE 3 Converting from Degrees to Radians

a. $135° = (135 \text{ deg})\left(\dfrac{\pi \text{ rad}}{180 \text{ deg}}\right) = \dfrac{3\pi}{4} \text{ radians}$ Multiply by $\dfrac{\pi \text{ rad}}{180°}$.

b. $540° = (540 \text{ deg})\left(\dfrac{\pi \text{ rad}}{180 \text{ deg}}\right) = 3\pi \text{ radians}$ Multiply by $\dfrac{\pi \text{ rad}}{180°}$.

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com.*

Rewrite each angle in radian measure as a multiple of π. (Do not use a calculator.)

a. $\theta = 60°$ **b.** $\theta = 320°$

EXAMPLE 4 Converting from Radians to Degrees

a. $-\dfrac{\pi}{2} \text{ rad} = \left(-\dfrac{\pi}{2} \text{ rad}\right)\left(\dfrac{180 \text{ deg}}{\pi \text{ rad}}\right) = -90°$ Multiply by $\dfrac{180°}{\pi \text{ rad}}$.

b. $2 \text{ rad} = (2 \text{ rad})\left(\dfrac{180 \text{ deg}}{\pi \text{ rad}}\right) = \dfrac{360°}{\pi} \approx 114.59°$ Multiply by $\dfrac{180°}{\pi \text{ rad}}$.

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com.*

Rewrite each angle in degree measure. (Do not use a calculator.)

a. $\pi/6$ **b.** $5\pi/3$

Applications

The *radian measure* formula, $\theta = s/r$, can be used to measure arc length along a circle.

Arc Length

For a circle of radius r, a central angle θ intercepts an arc of length s given by

$s = r\theta$ Length of circular arc

where θ is measured in radians. Note that if $r = 1$, then $s = \theta$, and the radian measure of θ equals the arc length.

EXAMPLE 5 Finding Arc Length

A circle has a radius of 4 inches. Find the length of the arc intercepted by a central angle of 240°, as shown in Figure 4.12.

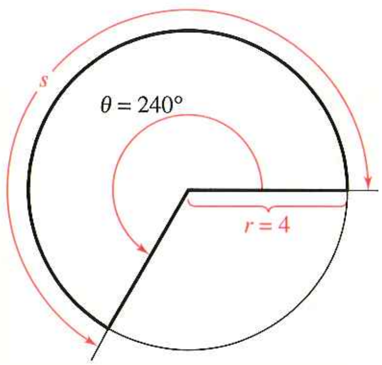


Figure 4.12

Solution To use the formula $s = r\theta$, first convert 240° to radian measure.

$$240° = (240 \text{ deg})\left(\frac{\pi \text{ rad}}{180 \text{ deg}}\right)$$

$$= \frac{4\pi}{3} \text{ radians}$$

Then, using a radius of $r = 4$ inches, you can find the arc length to be

$$s = r\theta$$

$$= 4\left(\frac{4\pi}{3}\right)$$

$$\approx 16.76 \text{ inches.}$$

Note that the units for r determine the units for $r\theta$ because θ is given in radian measure, which has no units.

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com.*

A circle has a radius of 27 inches. Find the length of the arc intercepted by a central angle of 160°.

REMARK Linear speed measures how fast the particle moves, and angular speed measures how fast the angle changes. By dividing each side of the formula for arc length by t, you can establish a relationship between linear speed v and angular speed ω, as shown.

$$s = r\theta$$

$$\frac{s}{t} = \frac{r\theta}{t}$$

$$v = r\omega$$

The formula for the length of a circular arc can help you analyze the motion of a particle moving at a *constant speed* along a circular path.

Linear and Angular Speeds

Consider a particle moving at a constant speed along a circular arc of radius r. If s is the length of the arc traveled in time t, then the **linear speed** v of the particle is

$$\text{Linear speed } v = \frac{\text{arc length}}{\text{time}} = \frac{s}{t}.$$

Moreover, if θ is the angle (in radian measure) corresponding to the arc length s, then the **angular speed** ω (the lowercase Greek letter omega) of the particle is

$$\text{Angular speed } \omega = \frac{\text{central angle}}{\text{time}} = \frac{\theta}{t}.$$

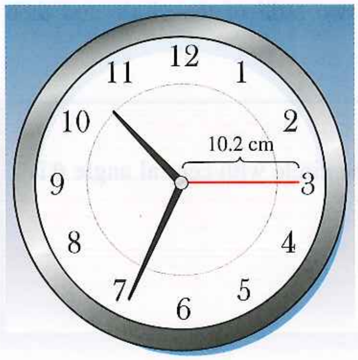


Figure 4.13

EXAMPLE 6 Finding Linear Speed

The second hand of a clock is 10.2 centimeters long, as shown in Figure 4.13. Find the linear speed of the tip of this second hand as it passes around the clock face.

Solution In one revolution, the arc length traveled is

$$s = 2\pi r$$
$$= 2\pi(10.2) \quad \text{Substitute for } r.$$
$$= 20.4\pi \text{ centimeters.}$$

The time required for the second hand to travel this distance is

$$t = 1 \text{ minute}$$
$$= 60 \text{ seconds.}$$

So, the linear speed of the tip of the second hand is

$$\text{Linear speed} = \frac{s}{t}$$
$$= \frac{20.4\pi \text{ centimeters}}{60 \text{ seconds}}$$
$$\approx 1.068 \text{ centimeters per second.}$$

✓ **Checkpoint** Audio-video solution in English & Spanish at LarsonPrecalculus.com.

The second hand of a clock is 8 centimeters long. Find the linear speed of the tip of this second hand as it passes around the clock face.

Figure 4.14

EXAMPLE 7 Finding Angular and Linear Speeds

The blades of a wind turbine are 116 feet long (see Figure 4.14). The propeller rotates at 15 revolutions per minute.

a. Find the angular speed of the propeller in radians per minute.

b. Find the linear speed of the tips of the blades.

Solution

a. Because each revolution generates 2π radians, it follows that the propeller turns

$$(15)(2\pi) = 30\pi \text{ radians per minute.}$$

In other words, the angular speed is

$$\text{Angular speed} = \frac{\theta}{t} = \frac{30\pi \text{ radians}}{1 \text{ minute}} = 30\pi \text{ radians per minute.}$$

b. The linear speed is

$$\text{Linear speed} = \frac{s}{t} = \frac{r\theta}{t} = \frac{(116)(30\pi) \text{ feet}}{1 \text{ minute}} \approx 10{,}933 \text{ feet per minute.}$$

✓ **Checkpoint** Audio-video solution in English & Spanish at LarsonPrecalculus.com.

The circular blade on a saw rotates at 2400 revolutions per minute.

a. Find the angular speed of the blade in radians per minute.

b. The blade has a radius of 4 inches. Find the linear speed of a blade tip.

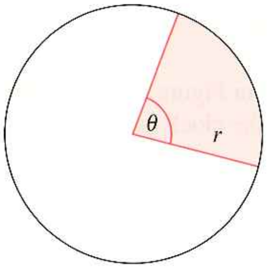


Figure 4.15

A **sector** of a circle is the region bounded by two radii of the circle and their intercepted arc (see Figure 4.15).

Area of a Sector of a Circle

For a circle of radius r, the area A of a sector of the circle with central angle θ is

$$A = \frac{1}{2}r^2\theta$$

where θ is measured in radians.

EXAMPLE 8 Area of a Sector of a Circle

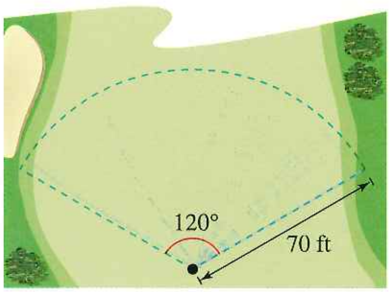


Figure 4.16

A sprinkler on a golf course fairway sprays water over a distance of 70 feet and rotates through an angle of 120° (see Figure 4.16). Find the area of the fairway watered by the sprinkler.

Solution

First convert 120° to radian measure as follows.

$$\theta = 120^\circ$$

$$= (120 \text{ deg})\left(\frac{\pi \text{ rad}}{180 \text{ deg}}\right) \qquad \text{Multiply by } \frac{\pi \text{ rad}}{180^\circ}.$$

$$= \frac{2\pi}{3} \text{ radians}$$

Then, using $\theta = 2\pi/3$ and $r = 70$, the area is

$$A = \frac{1}{2}r^2\theta \qquad \text{Formula for the area of a sector of a circle}$$

$$= \frac{1}{2}(70)^2\left(\frac{2\pi}{3}\right) \qquad \text{Substitute for } r \text{ and } \theta.$$

$$= \frac{4900\pi}{3} \qquad \text{Multiply.}$$

$$\approx 5131 \text{ square feet.} \qquad \text{Simplify.}$$

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com.*

A sprinkler sprays water over a distance of 40 feet and rotates through an angle of 80°. Find the area watered by the sprinkler.

Summarize (Section 4.1)

1. Describe an angle *(page 262)*.
2. Describe how to determine the measure of an angle using radians *(page 263)*. For examples involving radian measure, see Examples 1 and 2.
3. Describe how to determine the measure of an angle using degrees *(page 265)*. For examples involving degree measure, see Examples 3 and 4.
4. Describe examples of how to use angles to model and solve real-life problems *(pages 266–268, Examples 5–8)*.

4.1 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary: Fill in the blanks.

1. Two angles that have the same initial and terminal sides are ________.

2. One ________ is the measure of a central angle that intercepts an arc equal to the radius of the circle.

3. Two positive angles that have a sum of $\pi/2$ are ________ angles, whereas two positive angles that have a sum of π are ________ angles.

4. The angle measure that is equivalent to a rotation of $\frac{1}{360}$ of a complete revolution about an angle's vertex is one ________.

5. The ________ speed of a particle is the ratio of the arc length to the time traveled, and the ________ speed of a particle is the ratio of the central angle to the time traveled.

6. The area A of a sector of a circle with radius r and central angle θ, where θ is measured in radians, is given by the formula ________.

Skills and Applications

Estimating an Angle In Exercises 7–10, estimate the angle to the nearest one-half radian.

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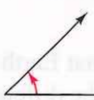

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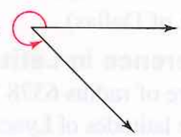

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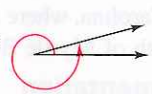


Determining Quadrants In Exercises 11 and 12, determine the quadrant in which each angle lies.

11. (a) $\frac{\pi}{4}$ (b) $\frac{5\pi}{4}$ **12.** (a) $-\frac{\pi}{6}$ (b) $-\frac{11\pi}{9}$

Sketching Angles In Exercises 13 and 14, sketch each angle in standard position.

13. (a) $\frac{\pi}{3}$ (b) $-\frac{2\pi}{3}$ **14.** (a) $\frac{5\pi}{2}$ (b) 4

Finding Coterminal Angles In Exercises 15 and 16, determine two coterminal angles (one positive and one negative) for each angle. Give your answers in radians.

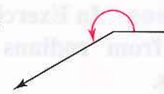
15. (a) $\frac{\pi}{6}$ (b) $\frac{7\pi}{6}$ **16.** (a) $\frac{2\pi}{3}$ (b) $-\frac{9\pi}{4}$

Complementary and Supplementary Angles In Exercises 17–20, find (if possible) the complement and the supplement of each angle.

17. (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ **18.** (a) $\frac{\pi}{12}$ (b) $\frac{11\pi}{12}$

19. (a) 1 (b) 2 **20.** (a) 3 (b) 1.5

Estimating an Angle In Exercises 21–24, estimate the number of degrees in the angle.

21.


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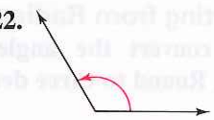

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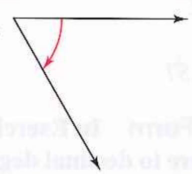

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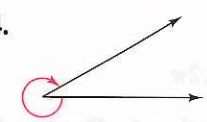


Determining Quadrants In Exercises 25 and 26, determine the quadrant in which each angle lies.

25. (a) 130° (b) 8.3°

26. (a) −132° 50′ (b) −3.4°

Sketching Angles In Exercises 27 and 28, sketch each angle in standard position.

27. (a) 270° (b) 120°

28. (a) −135° (b) −750°

Finding Coterminal Angles In Exercises 29 and 30, determine two coterminal angles (one positive and one negative) for each angle. Give your answers in degrees.

29. (a) 45° (b) −36°

30. (a) 120° (b) −420°

Complementary and Supplementary Angles In Exercises 31–34, find (if possible) the complement and the supplement of each angle.

31. (a) 18° (b) 85° **32.** (a) 46° (b) 93°

33. (a) 150° (b) 79° **34.** (a) 130° (b) 170°

Converting from Degrees to Radians **In Exercises 35 and 36, rewrite each angle in radian measure as a multiple of π. (Do not use a calculator.)**

35. (a) $120°$ (b) $-20°$

36. (a) $-60°$ (b) $144°$

Converting from Radians to Degrees **In Exercises 37 and 38, rewrite each angle in degree measure. (Do not use a calculator.)**

37. (a) $\frac{3\pi}{2}$ (b) $\frac{7\pi}{6}$

38. (a) $-\frac{7\pi}{12}$ (b) $\frac{5\pi}{4}$

Converting from Degrees to Radians **In Exercises 39–42, convert the angle measure from degrees to radians. Round to three decimal places.**

39. $45°$ **40.** $-48.27°$

41. $0.54°$ **42.** $345°$

Converting from Radians to Degrees **In Exercises 43–46, convert the angle measure from radians to degrees. Round to three decimal places.**

43. $\frac{5\pi}{11}$ **44.** $\frac{15\pi}{8}$

45. -4.2π **46.** -0.57

Converting to Decimal Degree Form **In Exercises 47 and 48, convert each angle measure to decimal degree form without using a calculator. Then check your answers using a calculator.**

47. (a) $54°\,45'$ (b) $-128°\,30'$

48. (a) $-135°\,36''$ (b) $-408°\,16'20''$

Converting to D°M′S″ Form **In Exercises 49 and 50, convert each angle measure to degrees, minutes, and seconds without using a calculator. Then check your answers using a calculator.**

49. (a) $240.6°$ (b) $-145.8°$

50. (a) $-345.12°$ (b) $-3.58°$

Finding Arc Length **In Exercises 51 and 52, find the length of the arc on a circle of radius r intercepted by a central angle θ.**

51. $r = 15$ inches, $\theta = 120°$

52. $r = 3$ meters, $\theta = 150°$

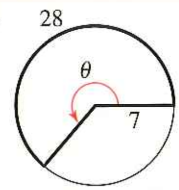
Finding the Central Angle **In Exercises 53 and 54, find the radian measure of the central angle of a circle of radius r that intercepts an arc of length s.**

53. $r = 80$ kilometers, $s = 150$ kilometers

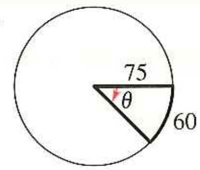
54. $r = 14$ feet, $s = 8$ feet

Finding an Angle **In Exercises 55 and 56, use the given arc length and radius to find the angle θ (in radians).**

55.



56.



Area of a Sector of a Circle **In Exercises 57 and 58, find the area of the sector of a circle of radius r and central angle θ.**

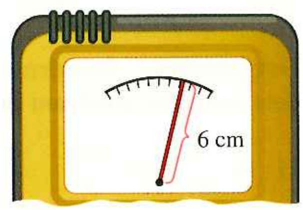
57. $r = 12$ millimeters, $\theta = \frac{\pi}{4}$

58. $r = 2.5$ feet, $\theta = 225°$

59. Distance Between Cities Find the distance between Dallas, Texas, whose latitude is $32°\,47'\,39''$N, and Omaha, Nebraska, whose latitude is $41°\,15'\,50''$N. Assume that Earth is a sphere of radius 4000 miles and that the cities are on the same longitude (Omaha is due north of Dallas).

60. Difference in Latitudes Assuming that Earth is a sphere of radius 6378 kilometers, what is the difference in the latitudes of Lynchburg, Virginia, and Myrtle Beach, South Carolina, where Lynchburg is about 400 kilometers due north of Myrtle Beach?

61. Instrumentation The pointer on a voltmeter is 6 centimeters in length (see figure). Find the number of degrees through which the pointer rotates when it moves 2.5 centimeters on the scale.



62. Linear Speed A satellite in a circular orbit 1250 kilometers above Earth makes one complete revolution every 110 minutes. Assuming that Earth is a sphere of radius 6378 kilometers, what is the linear speed (in kilometers per minute) of the satellite?

63. Angular and Linear Speeds The circular blade on a saw rotates at 5000 revolutions per minute.

(a) Find the angular speed of the blade in radians per minute.

(b) The blade has a diameter of $7\frac{1}{4}$ inches. Find the linear speed of a blade tip.

64. Angular and Linear Speeds A carousel with a 50-foot diameter makes 4 revolutions per minute.

(a) Find the angular speed of the carousel in radians per minute.

(b) Find the linear speed (in feet per minute) of the platform rim of the carousel.

65. Angular and Linear Speeds A DVD is approximately 12 centimeters in diameter. The drive motor of the DVD player rotates between 200 and 500 revolutions per minute, depending on what track is being read.

(a) Find an interval for the angular speed of the DVD as it rotates.

(b) Find an interval for the linear speed of a point on the outermost track as the DVD rotates.

66. Angular Speed A car is moving at a rate of 65 miles per hour, and the diameter of its wheels is 2 feet.

(a) Find the number of revolutions per minute the wheels are rotating.

(b) Find the angular speed of the wheels in radians per minute.

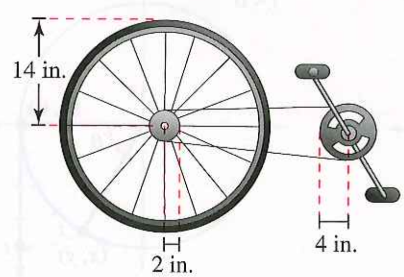
67. Linear and Angular Speeds A computerized spin balance machine rotates a 25-inch-diameter tire at 480 revolutions per minute.

(a) Find the road speed (in miles per hour) at which the tire is being balanced.

(b) At what rate should the spin balance machine be set so that the tire is being tested for 55 miles per hour?

68. Speed of a Bicycle

The radii of the pedal sprocket, the wheel sprocket, and the wheel of the bicycle in the figure are 4 inches, 2 inches, and 14 inches, respectively. A cyclist is pedaling at a rate of 1 revolution per second.



(a) Find the speed of the bicycle in feet per second and miles per hour.

(b) Use your result from part (a) to write a function for the distance d (in miles) a cyclist travels in terms of the number n of revolutions of the pedal sprocket.

(c) Write a function for the distance d (in miles) a cyclist travels in terms of the time t (in seconds). Compare this function with the function from part (b).

69. Area A sprinkler on a golf green sprays water over a distance of 15 meters and rotates through an angle of 140°. Draw a diagram that shows the region that the sprinkler can irrigate. Find the area of the region.

70. Area A car's rear windshield wiper rotates 125°. The total length of the wiper mechanism is 25 inches and wipes the windshield over a distance of 14 inches. Find the area covered by the wiper.

Exploration

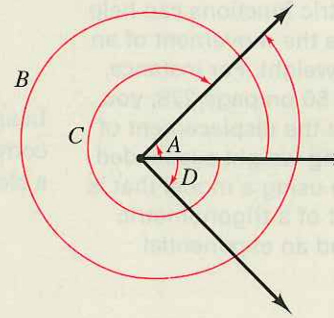
True or False? **In Exercises 71–73, determine whether the statement is true or false. Justify your answer.**

71. A measurement of 4 radians corresponds to two complete revolutions from the initial side to the terminal side of an angle.

72. The difference between the measures of two coterminal angles is always a multiple of 360° when expressed in degrees and is always a multiple of 2π radians when expressed in radians.

73. An angle that measures $-1260°$ lies in Quadrant III.

74. HOW DO YOU SEE IT? Determine which angles in the figure are coterminal angles with angle A. Explain your reasoning.



75. Think About It A fan motor turns at a given angular speed. How does the speed of the tips of the blades change when a fan of greater diameter is on the motor? Explain.

76. Think About It Is a degree or a radian the greater unit of measure? Explain.

77. Writing When the radius of a circle increases and the magnitude of a central angle is constant, how does the length of the intercepted arc change? Explain your reasoning.

78. Proof Prove that the area of a circular sector of radius r with central angle θ is

$$A = \frac{1}{2}\theta r^2$$

where θ is measured in radians.

4.2 Trigonometric Functions: The Unit Circle

Trigonometric functions can help you analyze the movement of an oscillating weight. For instance, in Exercise 50 on page 278, you will analyze the displacement of an oscillating weight suspended by a spring using a model that is the product of a trigonometric function and an exponential function.

- **Identify a unit circle and describe its relationship to real numbers.**
- **Evaluate trigonometric functions using the unit circle.**
- **Use domain and period to evaluate sine and cosine functions, and use a calculator to evaluate trigonometric functions.**

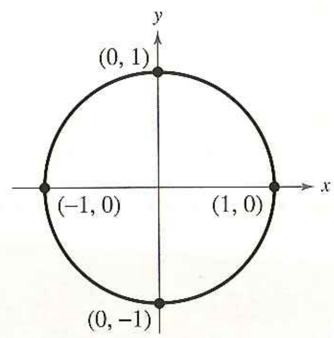
The Unit Circle

The two historical perspectives of trigonometry incorporate different methods for introducing the trigonometric functions. One such perspective follows and is based on the unit circle.

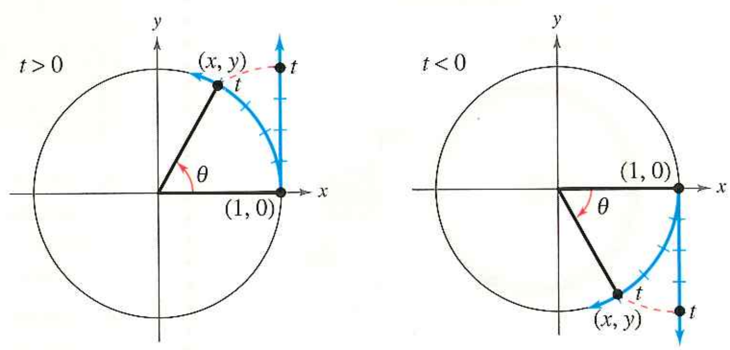
Consider the **unit circle** given by

$$x^2 + y^2 = 1 \qquad \text{Unit circle}$$

as shown below.



Imagine wrapping the real number line around this circle, with positive numbers corresponding to a counterclockwise wrapping and negative numbers corresponding to a clockwise wrapping, as shown below.



As the real number line wraps around the unit circle, each real number t corresponds to a point (x, y) on the circle. For example, the real number 0 corresponds to the point $(1, 0)$. Moreover, because the unit circle has a circumference of 2π, the real number 2π also corresponds to the point $(1, 0)$.

In general, each real number t also corresponds to a central angle θ (in standard position) whose radian measure is t. With this interpretation of t, the arc length formula

$$s = r\theta \quad (\text{with } r = 1)$$

indicates that the real number t is the (directional) length of the arc intercepted by the angle θ, given in radians.

Richard Megna/Fundamental Photographs