CHAPTER 4
Trigonometry

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Section 4.1 Radian and Degree Measure

1. coterminal

2. radian

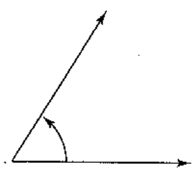
3. complementary; supplementary

4. degree

5. linear; angular

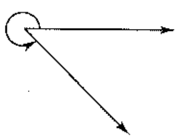
6. $A = \frac{1}{2}r^2\theta$

7.



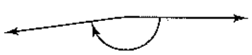
The angle shown is approximately 1 radian.

8.



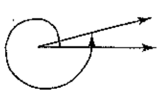
The angle shown is approximately 5.5 radians.

9.



The angle shown is approximately -3 radians.

10.



The angle shown is approximately 6.5 radians.

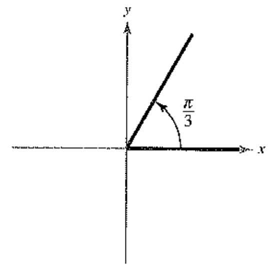
11. (a) Because $0 < \frac{\pi}{4} < \frac{\pi}{2}, \frac{\pi}{4}$ lies in Quadrant I.

(b) Because $\pi < \frac{5\pi}{4} < \frac{3\pi}{2}, \frac{5\pi}{4}$ lies in Quadrant III.

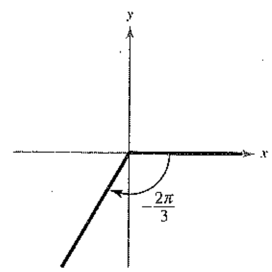
12. (a) Because $-\frac{\pi}{2} < -\frac{\pi}{6} < 0, -\frac{\pi}{6}$ lies in Quadrant IV.

(b) Because $-\frac{3\pi}{2} < -\frac{11\pi}{9} < -\pi, -\frac{11\pi}{9}$ lies in Quadrant II.

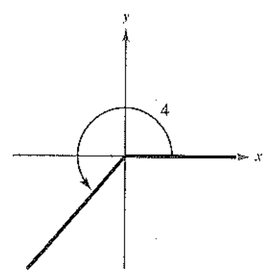
13. (a) $\frac{\pi}{3}$



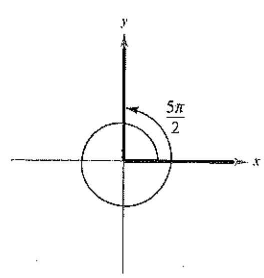
(b) $-\frac{2\pi}{3}$



14. (a) 4



(b) $\frac{5\pi}{2}$



15. (a) $\frac{\pi}{6} + 2\pi = \frac{13\pi}{6}$ $\frac{\pi}{6} - 2\pi = -\frac{11\pi}{6}$

(b) $\frac{7\pi}{6} + 2\pi = \frac{19\pi}{6}$ $\frac{7\pi}{6} - 2\pi = -\frac{5\pi}{6}$

16. *Sample answers:*

(a) $\frac{2\pi}{3} + 2\pi = \frac{8\pi}{3}$

$\frac{2\pi}{3} - 2\pi = -\frac{4\pi}{3}$

(b) $-\frac{9\pi}{4} + 2\pi = -\frac{\pi}{4}$

$-\frac{9\pi}{4} + 4\pi = \frac{7\pi}{4}$

17. (a) Complement: $\frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$

Supplement: $\pi - \frac{\pi}{3} = \frac{2\pi}{3}$

(b) Complement: $\frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

Supplement: $\pi - \frac{\pi}{4} = \frac{3\pi}{4}$

18. (a) Complement: $\frac{\pi}{2} - \frac{\pi}{12} = \frac{5\pi}{12}$

Supplement: $\pi - \frac{\pi}{12} = \frac{11\pi}{12}$

(b) Complement: Not possible, $\frac{11\pi}{12}$ is greater than $\frac{\pi}{2}$.

Supplement: $\pi - \frac{11\pi}{12} = \frac{\pi}{12}$

19. (a) Complement: $\frac{\pi}{2} - 1 \approx 0.57$

Supplement: $\pi - 1 \approx 2.14$

(b) Complement: Not possible, 2 is greater than $\frac{\pi}{2}$.

Supplement: $\pi - 2 \approx 1.14$

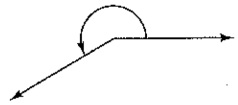
20. (a) Complement: Not possible, 3 is greater than $\frac{\pi}{2}$.

Supplement: $\pi - 3 \approx 0.14$

(b) Complement: $\frac{\pi}{2} - 1.5 \approx 0.07$

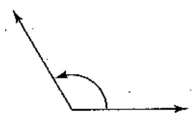
Supplement: $\pi - 1.5 \approx 1.64$

21.



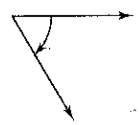
The angle shown is approximately $210°$.

22.



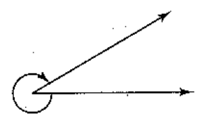
The angle shown is approximately $120°$.

23.



The angle shown is approximately $-60°$.

24.



The angle shown is approximately $-330°$.

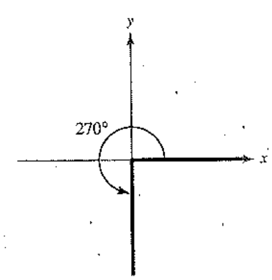
25. (a) Because $90° < 130° < 180°, 130°$ lies in Quadrant II.

(b) Because $0° < 8.3° < 90°, 8.3°$ lies in Quadrant I.

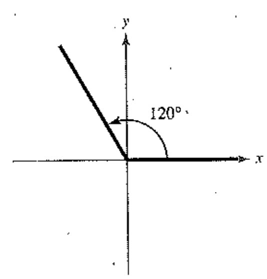
26. (a) Because $-180° < -132° 50' < -90°, -132° 50'$ lies in Quadrant III.

(b) Because $-90° < -3.4° < 0°, -3.4°$ lies in Quadrant IV.

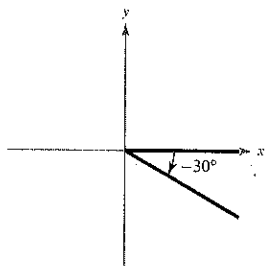
27. (a) $270°$



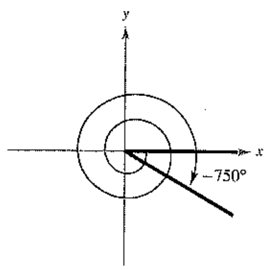
(b) $120°$



28. (a) $-30°$



(b) $-750°$



29. *Sample answers:*

(a) $45° + 360° = 405°$

$45° - 360° = -315°$

(b) $-36° + 360° = 324°$

$-36° - 360° = -396°$

30. *Sample answers:*

(a) $120° + 360° = 480°$

$120° - 360° = -240°$

(b) $-420° + 720° = 300°$

$-420° + 360° = -60°$

31. (a) Complement: $90° - 18° = 72°$

Supplement: $180° - 18° = 162°$

(b) Complement: $90° - 85° = 5°$

Supplement: $180° - 85° = 95°$

32. (a) Complement: $90° - 46° = 44°$

Supplement: $180° - 46° = 134°$

(b) Complement: Not possible, $93°$ is greater than $90°$.

Supplement: $180° - 93° = 87°$

33. (a) Complement: Not possible, $150°$ is greater than $90°$.

Supplement: $180° - 150° = 30°$

(b) Complement: $90° - 79° = 11°$

Supplement: $180° - 79° = 101°$

34. (a) Complement: Not possible, $130°$ is greater than $90°$.

Supplement: $180° - 130° = 50°$

(b) Complement: Not possible, $170°$ is greater than $90°$.

Supplement: $180° - 170° = 10°$

35. (a) $315° = 315°\left(\frac{\pi}{180°}\right) = \frac{7\pi}{4}$

(b) $-20° = -20\left(\frac{\pi}{180°}\right) = -\frac{\pi}{9}$

36. (a) $-60° = -60\left(\frac{\pi}{180°}\right) = -\frac{\pi}{3}$

(b) $144° = 144°\left(\frac{\pi}{180°}\right) = \frac{4\pi}{5}$

37. (a) $\frac{3\pi}{2} = \frac{3\pi}{2}\left(\frac{180°}{\pi}\right) = 270°$

(b) $\frac{7\pi}{6} = \frac{7\pi}{6}\left(\frac{180°}{\pi}\right) = 210°$

38. (a) $-\frac{7\pi}{12} = -\frac{7\pi}{12}\left(\frac{180°}{\pi}\right) = -105°$

(b) $-\frac{7\pi}{3} = -\frac{7\pi}{3}\left(\frac{180°}{\pi}\right) = -420°$

39. $45° = 45\left(\frac{\pi}{180°}\right) \approx 0.785$ radian

40. $-48.27° = -48.27°\left(\frac{\pi}{180°}\right) \approx -0.842$ radian

41. $345° = 345°\left(\frac{\pi}{180°}\right) \approx 6.021$ radians

42. $0.54° = 0.54°\left(\frac{\pi}{180°}\right) \approx 0.009$ radian

43. $\frac{5\pi}{11} = \frac{5\pi}{11}\left(\frac{180°}{\pi}\right) \approx 81.818°$

44. $\frac{15\pi}{8} = \frac{15\pi}{8}\left(\frac{180°}{\pi}\right) = 337.500°$

45. $-4.2\pi = -4.2\pi\left(\frac{180°}{\pi}\right) = -756.000°$

46. $-0.57 = -0.57\left(\frac{180°}{\pi}\right) \approx -32.659°$

47. (a) $54°45' = 54° + \left(\frac{45}{60}\right)^{\circ} = 54.75°$

(b) $-128°30' = -128° - \left(\frac{30}{60}\right)^{\circ} = -128.5°$

48. (a) $-135°36'' = -135° - \left(\frac{36}{3600}\right)^{\circ}$
$= -135° - 0.01° = -135.01°$

(b) $-408°16'20'' = -\left(408° + \left(\frac{16}{60}\right)^{\circ} + \left(\frac{20}{3600}\right)^{\circ}\right)$
$\approx -(408° + 0.2667° + 0.0056°)$
$\approx -408.272°$

49. (a) $240.6° = 240° + 0.6(60)' = 240°36'$

(b) $-145.8° = -[145° + 0.8(60')] = -145°48'$

50. (a) $-345.12° = -(345° + (0.12)(60'))$
$= -(345° + 7' + 0.2(60''))$
$= -345°7'12''$

(b) $-3.58° = -(3° + (0.58)(60'))$
$= -(3° + 34' + 0.8(60''))$
$= -3°34'48''$

51. $r = 15$ inches, $\theta = 120°$
$s = r\theta$
$s = 15(120°)\left(\frac{\pi}{180°}\right) = 10\pi$ inches
≈ 31.42 inches

52. $r = 9$ feet, $\theta = 150°$
$s = r\theta$
$s = 3(150°)\left(\frac{\pi}{180°}\right) = \frac{5\pi}{2}$ meters
≈ 7.85 meters

53. $r = 14$ feet, $s = 8$ feet
$s = r\theta$
$8 = 14\theta$
$\theta = \frac{8}{14} = \frac{4}{7}$ radian

54. $r = 80$ kilometers, $s = 150$ kilometers
$s = r\theta$
$150 = 80\theta$
$\theta = \frac{150}{80} = \frac{15}{8}$ radians

55. $s = r\theta$
$28 = 7\theta$
$\theta = 4$ radians

56. $s = r\theta$
$60 = 75\theta$
$\theta = \frac{60}{75} = \frac{4}{5}$ radian

Because the angle represented is clockwise, this angle is $-\frac{4}{5}$ radian.

57. $r = 12$ mm, $\theta = \frac{\pi}{4}$
$A = \frac{1}{2}r^2\theta = \frac{1}{2}(12)^2\left(\frac{\pi}{4}\right)$
$= 18\pi \text{ mm}^2$
$\approx 56.55 \text{ mm}^2$

58. $A = \frac{1}{2}r^2\theta$
$A = \frac{1}{2}(2.5)^2(225)\left(\frac{\pi}{180}\right)$
≈ 12.27 square feet

59. $\theta = 41°15'50'' - 32°47'39''$
$\approx 8.46972° \approx 0.14782$ radian
$s = r\theta \approx 4000(0.14782) \approx 591.3$ miles

60. $s = r\theta$
$400 = 6378\theta$
$\frac{400}{6378} = \theta$
$0.063 \approx \theta$

The difference in latitude is about 0.063 radian $\approx 3.59°$.

61. $s = r\theta$
$2.5 = 6\theta$
$\theta = \frac{2.5}{6} = \frac{25}{60} = \frac{5}{12}$ radian

62. The satellite makes one revolution per 110 minutes. The arc length the satellite travels in one revolution is
$s = r\theta \Rightarrow s = (6378 + 1250)(2\pi) = 47{,}928.14$ kilometers.

Therefore its linear speed is

$$\frac{47{,}928.14 \text{ kilometers}}{110 \text{ minutes}} \approx 435.7 \text{ km/min.}$$

63. diameter $= 7\frac{1}{4} = \frac{29}{4}$ inches

radius $= \frac{1}{2}$ diameter $= \frac{1}{2}\left(\frac{29}{4}\right) = \frac{29}{8}$ inches

(a) Angular speed $= \dfrac{(5000)(2\pi) \text{ radians}}{1 \text{ minute}}$

$= 10{,}000\pi$ radians per minute

$\approx 31{,}415.927$ radians per minute

(b) Linear speed $= \dfrac{\left(\frac{29}{8} \text{ in.}\right)\left(\frac{1 \text{ foot}}{12 \text{ in.}}\right)(5000)(2\pi)}{1 \text{ minute}}$

$= \dfrac{18{,}125\pi}{6}$ feet per minute

≈ 9490.23 feet per minute

64. (a) 4 rpm $= 4(2\pi)$ radians per minute

$= 8\pi$ radians per minute

≈ 25.13 radians per minute

(b) $r = 25$ ft

$\dfrac{r\theta}{t} = 200\pi$ feet per minute

Linear speed $\approx 25(25.13274)$ feet per minute

≈ 628.32 feet per minute

65. (a) $(200)(2\pi) \le$ Angular speed $\le (500)(2\pi)$ radians per minute

Interval: $[400\pi, 1000\pi]$ radians per minute

(b) $(6)(200)(2\pi) \le$ Linear speed $\le (6)(500)(2\pi)$ centimeters per minute

Interval: $[2400\pi, 6000\pi]$ centimeters per minute

66. diameter $= 2$ feet

radius $= \frac{1}{2}$ diameter $= \frac{1}{2}(2) = 1$ foot

(a) 65 miles per hour $= \dfrac{65(5280)}{60} = 5720$ feet per minute

The circumference of the tire is:

$C = 2\pi r = 2\pi(1) = 2\pi$ feet

The number of revolutions per minute:

$r = \dfrac{5720}{2\pi} \approx 910.37$ revolutions per minute

(b) Angular speed $= \dfrac{\theta}{t}$

$\theta = \dfrac{5720}{2\pi}\left(\dfrac{2\pi}{1}\right) = 5720$ radians

Angular speed $= \dfrac{5720 \text{ radians}}{1 \text{ minute}} = 5720$ radians per minute

67. diameter $= 25$ inches

radius $= \frac{1}{2}$ diameter $= \frac{1}{2}(25) = 12.5$ inches

(a) $12.5 \text{ in.} \times \dfrac{1 \text{ foot}}{12 \text{ in.}} \times \dfrac{1 \text{ mile}}{5280 \text{ feet}} \times \dfrac{480 \text{ rev}}{\text{min}} \times \dfrac{60 \text{ min}}{1 \text{ hr}} \times \dfrac{2\pi \text{ radians}}{\text{rev}}$

≈ 35.70 miles per hour

(b) $\dfrac{35.70 \text{ miles per hour}}{480 \text{ rev per minute}} = \dfrac{55 \text{ miles per hour}}{x \text{ rev per minute}}$

$35.70x = 26{,}400$

$x \approx 739.50$ revolutions per minute

68. (a) Arc length of larger sprocket in feet: $s = r\theta$

$$s = \frac{1}{3}(2\pi) = \frac{2\pi}{3} \text{ feet}$$

So, the chain moves $2\pi/3$ feet, as does the smaller rear sprocket. So, the angle θ of the smaller sprocket is $(r = 2 \text{ inches} = 2/12 \text{ feet})$.

$\theta = \frac{s}{r} = \frac{(2\pi)/3 \text{ feet}}{2/12 \text{ feet}} = 4\pi$ and the arc length of the tire in feet is:

$$s = \theta r$$

$$s = (4\pi)\left(\frac{14}{12}\right) = \frac{14\pi}{3} \text{ feet}$$

$$\text{Speed} = \frac{s}{t} = \frac{(14\pi)/3}{1 \text{ second}} = \frac{14\pi}{3} \text{ feet per second}$$

$$\frac{14\pi \text{ feet}}{3 \text{ seconds}} \times \frac{3600 \text{ seconds}}{1 \text{ hour}} \times \frac{1 \text{ mile}}{5280 \text{ feet}} \approx 10 \text{ miles per hour}$$

(b) Because the arc length of the tire is $(14\pi)/3$ feet and the cyclist is pedaling at a rate of one revolution per second, we have:

$$\text{Distance} = \left(\frac{14\pi}{3} \frac{\text{feet}}{\text{revolutions}}\right)\left(\frac{1 \text{ mile}}{5280 \text{ feet}}\right)(n \text{ revolutions}) = \frac{7\pi}{7920} n \text{ miles}$$

(c) Distance = Rate · Time

$$= \left(\frac{14\pi}{3} \text{ feet per second}\right)\left(\frac{1 \text{ mile}}{5280 \text{ feet}}\right)(t \text{ seconds}) = \frac{7\pi}{7920} t \text{ miles}$$

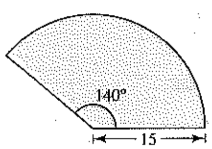
(d) The functions are both linear.

69. $A = \frac{1}{2}r^2\theta$

$= \frac{1}{2}(15)^2(140°)\left(\frac{\pi}{180°}\right)$

$= 87.5\pi$ square meters

≈ 274.89 square meters



70. $A = \frac{1}{2}\theta(R^2 - r^2)$

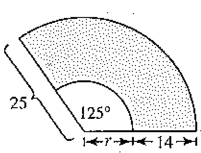
$R = 25$

$r = 25 - 14 = 11$

$A = \frac{1}{2}\left(\frac{125}{180}\right)\pi \cdot (25^2 - 11^2)$

$= 175\pi$

≈ 549.8 square inches



71. False. An angle measure of 4π radians corresponds to two complete revolutions from the initial side to the terminal side of an angle.

72. True. If α and β are coterminal angles, then $\alpha = \beta + n(360°)$ where n is an integer. The difference between α and β is $\alpha - \beta = n(360°) = 2\pi n$.

73. False. The terminal side of $-1260°$ lies on the negative x-axis.

74. Increases, because the linear speed is proportional to the radius.

75. $1 \text{ radian} = \left(\frac{180}{\pi}\right)^{\circ} \approx 57.3°$,

so one radian is much larger than one degree.

76. The arc length is increasing. In order for the angle θ to remain constant as the radius r increases, the arc length s must increase in proportion to r, as can be seen from the formula $s = r\theta$.

77. Since $s = r\theta$, then the rate of change of θ is 0 and so does $\frac{ds}{dt} = \theta\frac{dr}{dt}$.

That is, the arc length changes at a rate proportional to the rate of change of the radius and the proportionality constant is θ.

78. The area of a circle is $A = \pi r^2 \Rightarrow \pi = \frac{A}{r^2}$.

The circumference of a circle is $C = 2\pi r$.

$$C = 2\left(\frac{A}{r^2}\right)r$$

$$C = \frac{2A}{r}$$

$$\frac{Cr}{2} = A$$

For a sector, $C = s = r\theta$. So, $A = \frac{(r\theta)r}{2} = \frac{1}{2}\theta r^2$ for a sector.

Section 4.2 Trigonometric Functions: The Unit Circle

1. unit circle

2. periodic

3. period

4. odd; even

5. $x = \frac{12}{13}, y = \frac{5}{13}$

$\sin t = y = \frac{5}{13}$ $\quad\csc t = \frac{1}{y} = \frac{13}{5}$

$\cos t = x = \frac{12}{13}$ $\quad\sec t = \frac{1}{x} = \frac{13}{12}$

$\tan t = \frac{y}{x} = \frac{5}{12}$ $\quad\cot t = \frac{x}{y} = \frac{12}{5}$

6. $x = -\frac{8}{17}, y = \frac{15}{17}$

$\sin t = y = \frac{15}{17}$ $\quad\csc t = \frac{1}{y} = \frac{17}{15}$

$\cos t = x = -\frac{8}{17}$ $\quad\sec t = \frac{1}{x} = -\frac{17}{8}$

$\tan t = \frac{y}{x} = -\frac{15}{8}$ $\quad\cot t = \frac{x}{y} = -\frac{8}{15}$

7. $x = -\frac{4}{5}, y = -\frac{3}{5}$

$\sin t = y = -\frac{3}{5}$ $\quad\csc t = \frac{1}{y} = -\frac{5}{3}$

$\cos t = x = -\frac{4}{5}$ $\quad\sec t = \frac{1}{x} = -\frac{5}{4}$

$\tan t = \frac{y}{x} = \frac{3}{4}$ $\quad\cot t = \frac{x}{y} = \frac{4}{3}$

8. $x = \frac{12}{13}, y = -\frac{5}{13}$

$\sin t = y = -\frac{5}{13}$ $\quad\csc t = \frac{1}{y} = -\frac{13}{5}$

$\cos t = x = \frac{12}{13}$ $\quad\sec t = \frac{1}{x} = \frac{13}{12}$

$\tan t = \frac{y}{x} = -\frac{5}{12}$ $\quad\cot t = \frac{x}{y} = -\frac{12}{5}$

9. $t = \frac{\pi}{2}$ corresponds to the point $(x, y) = (0, 1)$.

10. $t = \frac{\pi}{4}$ corresponds to $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

11. $t = \frac{5\pi}{6}$ corresponds to the point $(x, y) = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$.

12. $t = \frac{4\pi}{3}$ corresponds to the point $(x, y) = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$.

13. $t = \frac{\pi}{4}$ corresponds to the point $(x, y) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$\sin\frac{\pi}{4} = y = \frac{\sqrt{2}}{2}$

$\cos\frac{\pi}{4} = x = \frac{\sqrt{2}}{2}$

$\tan\frac{\pi}{4} = \frac{y}{x} = 1$