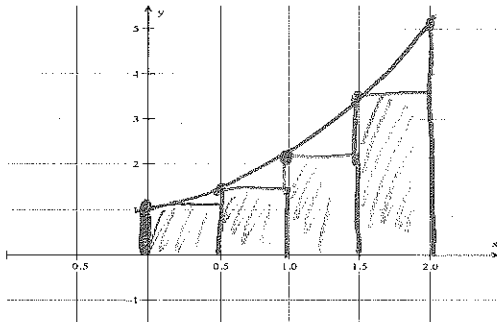


For #1-2 let $f(x) = x^2 + 1$

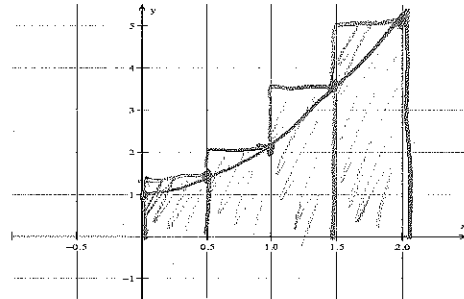
I. Find the left-hand and right hand sums on the interval $[0,2]$ with 4 intervals. Show all work. Then find the answer on your calculator.

1. Left - hand sum: 3.75

2. Right- hand sum: 5.75



$$.5(1) + .5\left(\frac{5}{4}\right) + .5(2) + .5(3.25)$$



$$.5(1.25) + .5(2) + .5(3.25) + .5(5)$$

Calculator: 4.67 or $\frac{14}{3}$

II. Find the indefinite integral of the following. Show all work!

3. $\int \sqrt[3]{y^2} dy = \int y^{2/3} dy = \frac{3}{5} y^{5/3} + C$

4. $\int (4 \csc^2 \theta) d\theta = 4 \int \csc^2 \theta d\theta = -4 \cot \theta + C$

5. $\int \frac{5x^5 - 2x^3 + 3x}{x} dx$

$$\int (5x^4 - 2x^2 + 3) dx$$

$$x^5 - \frac{2}{3}x^3 + 3x + C$$

6. If $f(2) = 11$ find $f(x)$ given $f'(x) = 5x - 1$

$$\int (5x - 1) dx = \frac{5}{2}x^2 - x + C = 11$$

$$\frac{5}{2}(2^2) - 2 + C = 11$$

$$10 - 2 + C = 11$$

$$8 + C = 11$$

$$C = 3$$

$$f(x) = 5x^2 - x + 3$$

III. Use the following values to evaluate #7-9

$$\int_2^4 x^3 dx = 40$$

$$\int_2^4 x dx = 4$$

$$\int_2^4 dx = 2$$

7. $\int_2^4 x^3 dx$
0

8. $\int_2^4 15 dx$
 $15 \int_2^4 dx = 15(2) = 30$

9. $\int_2^4 (x^3 + 4) dx$
 $\int_2^4 x^3 dx + 4 \int_2^4 dx$
 $40 + 4(2) = 48$

IV. Use the Fundamental Theorem of Calculus to evaluate the following definite integrals:

10. $\int_1^2 (\frac{1}{x^2} - \frac{1}{x^3}) dx$

$$\int_1^2 (x^{-2} - x^{-3}) dx$$

$$-x^{-1} + \frac{1}{2}x^{-2}$$

$$-\frac{1}{2} + \frac{1}{8} - (-1 + \frac{1}{2})$$

$$\frac{1}{8}$$

11. $\int_4^9 x\sqrt{x} dx$

$$\int_4^9 x(x^{1/2}) dx = \int_4^9 x^{3/2} dx$$

$$\frac{2}{5}x^{5/2}$$

$$\frac{2}{5}(243) - \frac{2}{5}(32)$$

$$423/5 = 84.4$$

12. $\int_{\pi/4}^{\pi/2} (\sec^2 x) dx$

$$\tan x \Big|_{\pi/4}^{\pi/2}$$

$$1 - 1 = 2$$

V. Find the indefinite integral using U-substitution:

13. $\int (1 - 2x^2)^3 (-4x) dx$

$u = 1 - 2x^2$
 $du = -4x dx$

$$\int u^3 du = \frac{1}{4}u^4$$

$$\frac{1}{4}(1 - 2x^2)^4 + C$$

14. $\int \frac{x}{\sqrt{x^2+9}} dx$

$u = x^2 + 9$
 $du = 2x dx$
 $\frac{1}{2} du = x dx$

$$\int x(x^2+9)^{1/2} dx$$

$$\frac{1}{2} \int u^{-1/2} du = 2 \cdot \frac{1}{2} u^{1/2} + C$$

$$(x^2+9)^{1/2} + C$$

15. $\int \cos 2x dx$

$u = 2x$
 $du = 2 dx$
 $\frac{1}{2} du = dx$

$$\int \cos u du$$

$$\frac{1}{2} \sin 2x + C$$

VI. Use the following to answer questions #16-23

$\frac{d}{dx} [\ln x] = \frac{1}{x}$	$\frac{d}{dx} [\ln u] = \frac{u'}{u}$	$\frac{d}{dx} \log_a u = \frac{1}{u \ln a} u'$
$\frac{d}{dx} e^x = e^x$	$\frac{d}{dx} e^u = e^u u'$	$\frac{d}{dx} a^u = a^u \ln a \cdot u'$
$\int \frac{1}{x} dx = \ln x + C$	$\int \frac{1}{u} du = \ln u + C$	$\int e^x dx = e^x + C$
$\int e^u du = e^u + C$	$\int a^u du = \frac{1}{\ln a} a^u + C$	

16. $y = 4^{3x}$. Find y'

$$4^{3x} (3) \ln 4$$

$$3 (4)^{3x} \ln 4$$

18. $y = x \ln x$ Find y'

$f = x$ $g = \ln x$
 $f' = 1$ $g' = \frac{1}{x}$

$$y' = 1 + \ln x$$

20. $\int e^{-x^4} (-4x^3) dx$

$u = -x^4$
 $du = -4x^3 dx$

$$\int e^u du = e^u + C = e^{-x^4} + C$$

17. $f(x) = \ln[x(x^2 + 1)^2]$ Find $f'(x)$

$$\ln x + 2 \ln(x^2 + 1)$$

$$\frac{1}{x} + \frac{2(2x)}{x^2 + 1} = \frac{1}{x} + \frac{4x}{x^2 + 1}$$

19. $y = \frac{e^x}{x}$ find y'

$f = e^x$ $g = x$
 $f' = e^x$ $g' = 1$

$$\frac{x e^x - e^x}{x^2}$$

21. $\int 3 \cdot 4^{3x} dx$

$u = 3x$
 $du = 3 dx$

$$\int 4^u du$$

$$\frac{1}{\ln 4} \cdot 4^{3x} + C$$

22. $\int \frac{2x^2+7x-3}{x-2} dx$ (hint: divide) $x-2 \overline{) 2x^2+7x-3}$

$\int 2x+11 dx + 19 \int \frac{1}{x-2}$

$x^2+11x+19 \ln|x-2| + C$

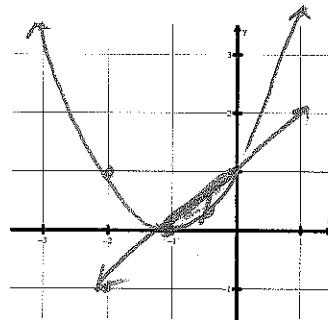
$\begin{array}{r} 2x+11 \text{ R } 19 \\ \underline{-(2x-4x)} \\ 11x-3 \\ \underline{-(11x-22)} \\ 19 \end{array}$

23. $\int 2^{\sin x} \cos x dx$ $u = \sin x$
 $du = \cos x dx$
 $\int 2^u du$
 $\frac{1}{\ln 2} 2^{\sin x} + C$

VII. Sketch the area bounded by the graphs of the functions and find the area of the region. Find the integral and show all work for full credit!

24. $f(x) = x^2 + 2x + 1$
 $g(x) = x + 1$

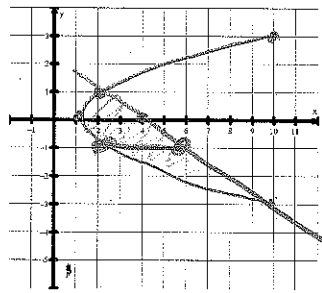
$\int_{-1}^0 (x+1) - (x^2+2x+1) dx$
 $\int_{-1}^0 (x+1-x^2-2x-1) dx = \int_{-1}^0 -x^2-x = \frac{1}{6}$



25. $x = y^2 + 1$
 $x = 4 - 2y$

$\int_{-3}^1 (4-2y) - (y^2+1) dy$
 $\int_{-3}^1 (-y^2-2y+3) dy = \frac{32}{3}$

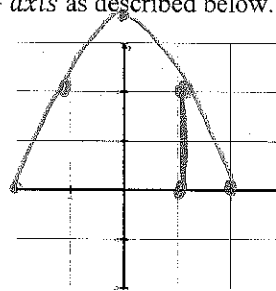
x	4-2y
4	4
6	-1
5	0
10	-3



26. Find the volume of the solid bounded by the graphs $y = 4 - x^2$ and the x -axis as described below. All cross-sections are perpendicular to the x -axis.

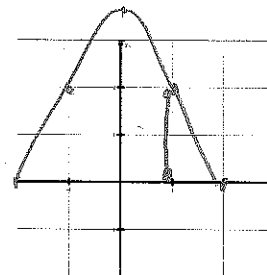
a) Cross sections are squares.

$\int_{-2}^2 (4-x^2)^2 dx = \frac{512}{15} = 34.13$



b) Cross-sections are rectangles with a height of x^2 .

$\int_{-2}^2 x^2(4-x^2) dx = \frac{128}{15} = 8.53$

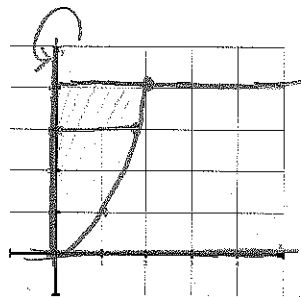


$\pi \int r^2$

27. Find the volume of the solid formed by revolving the region bounded by the graph(s) of the equation(s) about the y -axis using the disk method.

(a) $y = x^2, y = 0, y = 4, x = 0$

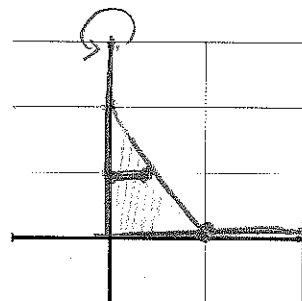
$\sqrt{y} = x$
 $\pi \int_0^4 (\sqrt{y})^2 dy = \pi \int_0^4 y dy = 8\pi = 25.13$



(b) $x = 1 - \frac{1}{2}y, x = 0, y = 0$

$\pi \int_0^2 (1 - \frac{1}{2}y)^2 dy = 2.09$

$X \mid Y$
 $1 \mid 0$
 $\frac{1}{2} \mid 2$

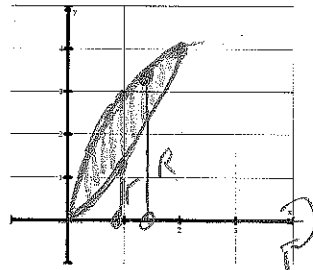


28. Find the volume of the solid formed by revolving the region bounded by the graph(s) of the equation(s) washer method.

(a) $y = x^2, y = 4x - x^2$ revolve about the x -axis

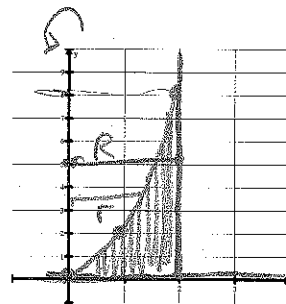
$\pi \int (R^2 - r^2)$

$\pi \int_0^2 (4x - x^2)^2 - (x^2)^2 dx = 33.51$



(b) $y = 2x^2, y = 0, x = 2$ revolve about the y -axis

$\sqrt{\frac{y}{2}} = x$
 $\pi \int_0^8 2^2 - (\sqrt{\frac{y}{2}})^2 dy = 50.27$



- (1) Left-sum: 3.75 (2) Right-Sum: 5.75, Calc: 4.67 (3) $\frac{3}{5}y^{\frac{5}{3}} + C$ (4) $-4 \cot \theta + C$ (5) $x^5 - \frac{2}{3}x^3 + 3x + C$ (6) $f(x) = \frac{5}{2}x^2 - x + 3$ (7) 0 (8) 30 (9) 48 (10) $\frac{1}{8}$ (11) $\frac{422}{5}$ (12) 2 (13) $\frac{1}{4}(1 - 2x^2)^4 + C$ (14) $\sqrt{x^2 + 9} + C$ (15) $\frac{1}{2} \sin 2x + C$ (16) $y' = 3(4^{3x}) \ln 4$ (17) $f' = \frac{1}{x} + \frac{4x}{x^2+1}$ (18) $y' = 1 + \ln x$ (19) $y' = \frac{xe^x - e^x}{x^2}$ (20) $e^{-x^4} + C$ (21) $\frac{4^{3x}}{\ln 4} + C$ (22) $x^2 + 11x + 19 \ln|x-2| + C$ (23) $\frac{2^{\sin x}}{\ln 2} + C$ (24) $\frac{1}{6}$ (25) $\frac{32}{3}$ (26a) $\frac{512}{15}$ (26b) $\frac{128}{15}$ (27a) 25.13 (27b) 2.09 (28a) 33.51 (28b) 50.27