

Spring Final Review

1. Given $f(x) = 4x$; $g(x) = 2x^2 - 3$, find $(g \circ f)(x)$.

$$\begin{aligned} g(f(x)) &= 2(4x)^2 - 3 \\ &= 2 \cdot 16x^2 - 3 \\ &= \boxed{32x^2 - 3} \end{aligned}$$

2. Given $f(x) = x^2 + 3x - 1$; $g(x) = 2x + 3$, find $(f \circ g)(x)$.

$$\begin{aligned} f(g(x)) &= (2x+3)^2 + 3(2x+3) - 1 \\ &= (4x^2 + 12x + 9) + (6x + 9) - 1 \\ &= \boxed{4x^2 + 18x + 17} \end{aligned}$$

3. Form a polynomial with zeros $-1, 1, 3$ and degree 3.

$$\begin{aligned} &(x+1)(x-1)(x-3) \\ &(x^2-1)(x-3) \\ &\boxed{x^3 - 3x^2 - x + 3} \end{aligned}$$

4. Form a polynomial with zeros $-2, 2, 3$ and degree 3.

$$\begin{aligned} &(x+2)(x-2)(x-3) \\ &(x^2-4)(x-3) \\ &\boxed{x^3 - 3x^2 - 4x + 12} \end{aligned}$$

5. Is $(x-1)$ a factor of $(2x^3 - x^2 + 2x - 3)$?

$$\begin{array}{r} 1 \overline{) 2 \ -1 \ 2 \ -3} \\ \underline{ 2 \ -1 \ 3} \\ 0 \end{array} \rightarrow \boxed{\text{YES}}$$

6. Is $(x+2)$ a factor of $(2x^3 - x^2 + 2x - 3)$?

$$\begin{array}{r} -2 \overline{) 2 \ -1 \ 2 \ -3} \\ \underline{ 2 \ -4 \ 10} \\ 12 \ -27 \end{array} \rightarrow \boxed{\text{No}}$$

7. Solve: $2x - 8 = \sqrt{x-1}$

$$\begin{aligned} (2x-8)^2 &= (\sqrt{x-1})^2 \\ 4x^2 - 32x + 64 &= x-1 \\ 4x^2 - 33x + 65 &= 0 \\ x &= \frac{33 \pm \sqrt{(-33)^2 - 4(4)(65)}}{2(4)} \end{aligned}$$

$$\boxed{x = 5, \frac{13}{4}}$$

8. Solve: $4 = \sqrt{x+1}$

$$\begin{aligned} 16 &= x+1 \\ \boxed{15} &= x \end{aligned}$$

17. Determine the zeros of $f(x) = 3x^3 - 6x^2 - 24x$.

$$0 = 3x(x^2 - 2x - 8)$$

$$0 = 3x(x-4)(x+2)$$

$$\boxed{x = 0, 4, -2}$$

18. Determine the zeros of $f(x) = 4x^3 - 8x^2 - 32x$.

$$0 = 4x(x^2 - 2x - 8)$$

$$0 = 4x(x-4)(x+2)$$

$$\boxed{x = 0, 4, -2}$$

19. Solve: $4^{x-1} = 64$

$$4^{x-1} = 4^3$$

$$x-1 = 3$$

$$\boxed{x = 4}$$

20. Solve: $2^{3x+1} = 128$

$$2^{3x+1} = 2^7$$

$$3x+1 = 7$$

$$\boxed{x = 2}$$

21. Evaluate: $\ln e^{16}$

$$\boxed{16}$$

22. Evaluate: $\log_{10} 10^{\pi}$

$$\boxed{\pi}$$

23. Write the exponential form of $\log_6 x = y$.

$$\boxed{6^y = x}$$

24. Write the exponential form of $\log_x y = 2$.

$$\boxed{x^2 = y}$$

25. Solve for x to the nearest hundredth: $\log x = 4.6561$

$$x = 10^{4.6561}$$

$$\boxed{x = 45300.19}$$

26. Solve for x to the nearest hundredth: $\ln x = 3.2517$

$$x = e^{3.2517}$$

$$\boxed{x = 25.83}$$

27. Solve for x to the nearest hundredth: $\log_8 3 = x$

$$x = \frac{\log 3}{\log 8}$$

$$\boxed{x = .53}$$

28. Solve for x to the nearest hundredth: $\log_4 9 = x$

$$x = \frac{\log 9}{\log 4}$$

$$\boxed{x = 1.58}$$

29. Solve: $\log_3(x+4) - \log_3(x+2) = 3$

$$\log_3\left(\frac{x+4}{x+2}\right) = 3 \rightarrow 27x + 54 = x + 4$$

$$26x = -50$$

$$x = \frac{-50}{26}$$

$$x = -\frac{25}{13}$$

31. Find the domain of $f(x) = \frac{4}{x+3}$

$$x \neq -3$$

30. Solve: $\log_2(x-3) - \log_2(x-4) = 1$

$$\log_2\left(\frac{x-3}{x-4}\right) = 1$$

$$2 = \frac{x-3}{x-4}$$

$$2x - 8 = x - 3$$

$$x = 5$$

32. Find the domain of $f(x) = \frac{1}{x-4}$

$$x \neq 4$$

33. Solve and write your answer in interval notation: $x^2 + 2x + 1 > 0$

$$(x+1)(x+1) > 0$$



PLUG IN -2: $(-2+1)(-2+1) = 1$

PLUG IN 0: $(0+1)(0+1) = 1$

$$(-\infty, -1) \cup (-1, \infty)$$

34. Solve and write your answer in interval notation: $2x^2 - 2x - 4 < 0$

$$2(x^2 - x - 2) < 0$$

$$2(x-2)(x+1) < 0$$



PLUG IN -2: $2(-2-2)(-2+1) = 8$

PLUG IN 0: $2(0-2)(0+1) = -4$

PLUG IN 3: $2(3-2)(3+1) = 8$

$$(-1, 2)$$

35. Give the equation of a hyperbola.

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

36. Give the equation of an ellipse.

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

37. Give the equation used to find the foci of an ellipse.

$$c^2 = a^2 - b^2$$

38. Give the equation used to find the foci of a hyperbola.

$$c^2 = a^2 + b^2$$

39. Determine the foci of the hyperbola: $\frac{x^2}{36} - \frac{y^2}{64} = 1$

CENTER: $(0,0)$

x-DIRECTION

$$c^2 = a^2 + b^2$$

$$c^2 = 36 + 64$$

$$c = 10$$

$$(10,0) \quad (-10,0)$$

40. Determine the foci of the ellipse: $\frac{x^2}{16} + \frac{y^2}{25} = 1$

CENTER: $(0,0)$

y-DIRECTION

$$c^2 = a^2 - b^2$$

$$c^2 = 25 - 16$$

$$c = 3$$

$$(0,3) \quad (0,-3)$$

41. Determine an equation in standard form of an ellipse with foci (3,2) and (-3,2), and a major axis of length 10.



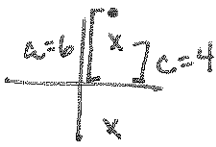
$$9 = 25 - b^2$$

$$b^2 = 16$$

CENTER: (0,2)

$$\frac{x^2}{25} + \frac{(y-2)^2}{16} = 1$$

42. Determine an equation in standard form of an ellipse with foci (3,4) and (3,-4), and a major axis of length 12.



$$16 = 36 - b^2$$

$$b^2 = 20$$

CENTER: (3,0)

$$\frac{(x-3)^2}{20} + \frac{y^2}{36} = 1$$

43. Given a hyperbola, the equations to find the asymptotes are:

$$y - k = \pm m(x - h)$$

44. Determine the center and radius of the circle: $x^2 + y^2 - 2x - 4y - 4 = 0$.

$$x^2 - 2x + 1 + y^2 - 4y + 4 = 4 + 1 + 4$$

$$(x-1)^2 + (y-2)^2 = 9$$

$$\text{CENTER: } (1, 2) \quad r = 3$$

45. Determine the center and radius of the circle: $x^2 + y^2 + 4x + 2y - 20 = 0$.

$$x^2 + 4x + 4 + y^2 + 2y + 1 = 20 + 4 + 1$$

$$(x+2)^2 + (y+1)^2 = 25$$

$$\text{CENTER: } (-2, -1) \quad r = 5$$

46. Identify the equation without completing the square: $4x^2 - 3y^2 - x + 2y = 0$.

HYPERBOLA

47. Identify the equation without completing the square: $2x^2 + 3y^2 - x + 2y = 0$.

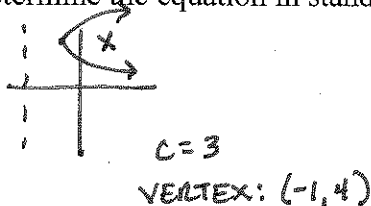
ELLIPSE

48. Determine the equation in standard form of a parabola with focus (-2,0) and directrix $x = 2$.



$$y^2 = -8x$$

49. Determine the equation in standard form of a parabola with focus (2,4) and directrix $x = -4$.



$$(y-4)^2 = 12(x+1)$$

50. Find an explicit formula for the sequence: $10, 9, \frac{81}{10}, \frac{729}{100}, \dots$

$$a_n = 10 \left(\frac{9}{10}\right)^{n-1}$$

$$\times \frac{9}{10}$$

51. Find an explicit formula for the sequence: $4, 7, 10, 13, \dots$

$$a_n = 4 + (n-1) \cdot 3$$

$$+3$$

52. Does the series converge; if so, to what value?

$$\sum_{k=1}^{\infty} 2 \left(\frac{1}{3}\right)^{k-1} \quad \boxed{\text{CONVERGES}} \quad \frac{2}{1-\frac{1}{3}} = \boxed{3}$$

53. Does the series converge; if so, to what value?

$$\sum_{k=1}^{\infty} 3 \left(\frac{4}{3}\right)^{k-1} \quad \boxed{\text{DIVERGES}}$$

54. Find the eighth term of the expansion of $(x+4y)^9$.

$$\begin{array}{cccccccc} 9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{array} \quad \left| \begin{array}{c} x^2 \\ (4y)^7 \\ (9) = 36 \end{array} \right| \quad \boxed{589824x^2y^7}$$

55. Decompose the fraction: $\frac{x}{(x-3)(x-2)}$

$$\frac{x}{(x-3)(x-2)} = \frac{A}{x-3} + \frac{B}{x-2}$$

$$x = A(x-2) + B(x-3)$$

PLUG IN 2: $2 = -B \rightarrow B = -2$

PLUG IN 3: $3 = A$

$$\frac{3}{x-3} + \frac{-2}{x-2}$$

56. Decompose the fraction: $\frac{3x+18}{x^2+5x+4} = \frac{3x+18}{(x+4)(x+1)}$

$$\frac{3x+18}{(x+4)(x+1)} = \frac{A}{x+4} + \frac{B}{x+1}$$

$$3x+18 = A(x+1) + B(x+4)$$

PLUG IN -1: $15 = 3B \rightarrow B = 5$

PLUG IN -4: $6 = -3A \rightarrow A = -2$

$$\frac{-2}{x+4} + \frac{5}{x+1}$$

57. Find the first derivative of $f(x) = 3x^2 + 2x + 5$.

$$f'(x) = \lim_{h \rightarrow 0} \frac{3(x+h)^2 + 2(x+h) + 5 - (3x^2 + 2x + 5)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{3x^2 + 6xh + 3h^2 + 2x + 2h + 5 - 3x^2 - 2x - 5}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{6xh + 3h^2 + 2h}{h}$$

$$\boxed{f'(x) = 6x + 2}$$

58. Find the equation of the line tangent to the graph of $f(x) = 2x^2 - 4x + 3$ at $P(1,1)$.

$$f'(x) = \lim_{h \rightarrow 0} \frac{2(x+h)^2 - 4(x+h) + 3 - (2x^2 - 4x + 3)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{2x^2 + 4xh + 2h^2 - 4x - 4h + 3 - 2x^2 + 4x - 3}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{4xh + 2h^2 - 4h}{h}$$

$$f'(x) = 4x - 4$$

$$f'(1) = 0$$

$$y - 1 = 0(x - 1) \rightarrow \boxed{y = 1}$$

59. Find $\lim_{x \rightarrow -2} (2x^4 - 3x^3 - x^2)$.

$$2(-2)^4 - 3(-2)^3 - (-2)^2 = 32 + 24 - 4 = \boxed{52}$$

60. Find $\lim_{x \rightarrow 6} (3x^2 - 4x - 2)$.

$$3(6)^2 - 4(6) - 2 = 108 - 24 - 2 = \boxed{82}$$